ECIT 2024

Abstracts

May 14, 2024

Contents

1 Anatoli F. Ivanov

1.1 Homtervals and infinitely many sinks for smooth interval maps

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This talk is based on my early and very first joint research effort with my teacher and PhD advisor Prof.Dr. O.M. Sharkovsky. It was an answer to a simply stated question by Prof. Szlenk of Poland whether smooth interval maps can have infinitely many sinks and wandering intervals. The answer was short and affirmative for C^∞-maps; it was published in 1983 in the Ukrainian Mathematical Journal [1].

We show that there exist C^∞-interval maps which have both infinitely many sinks with arbitrarily large periods and nontrivial homtervals (wandering intervals). A homterval is an interval on which all iterations of a map are homeomorphisms (monotone). Attracting cycles can demonstrate trivial examples of homtervals. Dynamics with nontrivial homtervals and infintitely many sinks are atypical; in particular, the limiting critical points in such dynamics must be flat (all derivatives are zero).

We also mention some other related results in papers [2, 3, 4].

References

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2 Andrey L. Shilnikov

2.1 Bifurcation structure of interval maps with orbits homoclinic to a saddle-focus

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We study homoclinic bifurcations in an interval map associated with a saddle-focus of (2, 1)-type in Z2-symmetric systems. Our study of this map reveals the homoclinic structure of the saddle-focus, with a bifurcation unfolding guided by the codimension-two Belyakov bifurcation. We consider three parameters of the map, corresponding to the saddle quantity, splitting parameter, and focal frequency of the smooth saddle-focus in a neighborhood of homoclinic bifurcations. We symbolically encode dynamics of the map in order to find stability windows and locate homoclinic bifurcation sets in a computationally efficient manner. The organization and possible shapes of homoclinic bifurcation curves in the parameter space are examined, taking into account the symmetry and discontinuity of the map. Sufficient conditions for stability and local symbolic constancy of the map are presented. This study furnishes insights into the structure of homoclinic bifurcations of the saddle-focus map, furthering comprehension of low-dimensional chaotic systems.

3 Antonio Garijo Real

3.1 Dynamics of the Secant map near a critical three cycle

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In this talk we give analytic proofs for the existence of transversal homoclinic points for a family of non globally smooth diffeomorphisms having the origin as a fixed point which come out as a truncated map governing the local dynamics of a critical period three cycle associated to the Secant map. Using Moser-Birkoff-Smale's Theorem, we prove that the boundary of the basin of attraction of the origin contains a Cantor-like invariant subset whose restricted dynamics is conjugated to the full shift of 2-symbols. This is a joint work with X. Jarque and E. Fontich.

4 Chaitanya Gopalakrishna

4.1 Iteration operators on the space of continuous self-maps

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The dynamical system of a continuous self-map is generated by iteration of the map; however, the iteration itself, being an operator on the space of continuous self-maps, may generate interesting dynamical behaviors. In this talk we discuss some dynamical properties of the iteration operators on the space of continuous self-maps of a locally compact Hausdorff space. The talk is based on our recent works [1, 2].

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5 Davide Radi

5.1 Bifurcation structures of a two-dimensional piecewise linear discontinuous map: Analysis of a cobweb model with regime-switching expectations

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We consider the bifurcations occurring in a two-dimensional piecewise-linear discontinuous map that describes the dynamics of a cobweb model in which firms rely on a regime switching expectation rule. In three different partitions of the phase plane, separated by two discontinuity lines, the map is defined by linear functions with the same Jacobian matrix, having two real eigenvalues, one of which is negative and one equal to 0. This leads to asymptotic dynamics that can belong to two or three critical lines. We show that when the basic fixed point is attracting, it may coexist with at most three attracting cycles. We have determined their existence regions, in the two-dimensional parameter plane, bounded by border collision bifurcation curves. At parameter values for which the basic fixed point is repelling, chaotic attractors may exist - either one that is symmetric with respect to the basic fixed point, or, if not symmetric, the symmetric one also exists. The homoclinic bifurcations of repelling cycles leading to the merging of chaotic attractors are commented by using the first return map on a suitable line. Moreover, four different kinds of homoclinic bifurcations of a saddle 2-cycle, leading to divergence of the generic trajectory, are determined.

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6 Dorota Głazowska

6.1 Subcommutativity of integrals similar to the integral quasi-arthmetic means

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Let $(X, \mathscr{L}, \lambda)$ and $(Y, \mathscr{M}, \mu)$ be finite non-degenerate measure spaces, $I \subseteq \mathbb{R}$ be a non-empty interval, and $f, g\colon I \to \mathbb{R}$ be continuous injections. We look for conditions on f and g under which the inequality

$$f^{-1}\left(\int_X f\left(g^{-1}\left(\int_Y g \circ h \, d\mu\right)\right) d\lambda\right) \le g^{-1}\left(\int_Y g\left(f^{-1}\left(\int_X f \circ h \, d\lambda\right)\right) d\mu\right) \tag{1}$$

is satisfied for every $h\colon X \times Y \to I$ in a suitable class of $\mathscr{L} \otimes \mathscr{M}$-measurable simple functions, taking for granted that each side of the above inequality is well defined. Notice that setting

$$\mathcal{M}_f(h) = f^{-1}\left(\int_X f \circ h \, d\lambda\right), \qquad \mathcal{M}_g(h) = g^{-1}\left(\int_Y g \circ h \, d\mu\right)$$

for the $\mathscr{L} \otimes \mathscr{M}$-measurable simple functions $h\colon X \times Y \to I$, the inequality (1) holds if

$$\mathcal{M}_f \circ \mathcal{M}_g \le \mathcal{M}_g \circ \mathcal{M}_f,$$

and it can be interpreted as the ”subcommutativity of the pair $(\mathcal{M}_f, \mathcal{M}_g)$” or ”supercommutativity of the pair $(\mathcal{M}_g, \mathcal{M}_f)$”.

Moreover, if $(X, \mathscr{L}, \lambda)$ and $(Y, \mathscr{M}, \mu)$ are probability spaces and the image $h(X \times Y)$ is contained in a compact subset of I for every test function h, which is especially the case when $h\colon X \times Y \to I$ is an $\mathscr{L} \otimes \mathscr{M}$-measurable simple function, then each side of the inequality (1) is well posed and can be interpreted as ”partially mixed integral quasi-arithmetic means”.

During the talk I am going to present results from joint work with Paolo Leonetti, Janusz Matkowski and Salvatore Tringali.

7 Eddy Kwessi

7.1 Information Theory in a Darwinian Evolution Population Dynamics

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Using information theory, we propose an estimation method for traits parameters in a Darwinian evolution model for species with one trait or multiple traits. We propose to estimate parameters by minimizing the relative information in a Darwinian evolution population model using either a classical gradient ascent or a stochastic gradient ascent. The proposed procedure is shown to be possible in a supervised or unsupervised learning environment, similarly to Boltzmann machines. Simulations are provided for illustration of the method.

8 Francisco Balibrea

8.1 O.M.Sharkovsky, friend and magister

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In this lecture I am dealing with the figure of O.M.Sharkovsky as a brilliant mathematician and a good friend. He passed away in 2022 in Kiev (Ukraine).

I will concentrate in part of his research: the introduction of his particular ordering of the natural numbers, his relevant theorem (well known currently in all the matematical community) its consequences in the theory and developing of *Topological Dynamical Systems*.

One of the consequences was the appearence of the *Combinatorial Dynamics* and the florishing of the *one dimension real dynamical systems*.

The lecture will have as a complementary item, the showing of some photos.

9 Gregory Derfel

9.1 On the balanced pantograph equation of mixed type

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We consider the balanced pantograph equation (BPE)

$$y'(x) + y(x) = \sum_{k=1}^{m} p_k y(a_k x),$$

where $a_k, p_k > 0$, and $\sum_{k=1}^{m} p_k = 1$. It is known that if $K = \sum_{k=1}^{m} p_k \ln a_k \leq 0$ then, under mild technical conditions, the BPE does not have bounded solutions that are not constant; whereas, if $K > 0$ such solutions exist. In the present paper we deal with a BPE of the *mixed type* i.e. $a_1 < 1 < a_m$, when $K > 0$. We prove that in this case the BPE has a non-constant solution y, such that $y(x) \sim cx^{-\sigma}$ as $x \to \infty$, where $c > 0$ and σ is the unique positive root of the characteristic equation $P(s) = 1 - \sum_{k=1}^{m} p_k a_k^{-s} = 0$. We also show that y is unique (up to a multiplicative constant) among solutions of the BPE that decay to zero as $x \to \infty$.

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10 Henrique Oliveira

10.1 Analysis of Synchronized States in Three Coupled Oscillators via Nonlinear Dynamics

Author(s): Henrique Oliveira, Emma D'Aniello and Jorge Buescu
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This talk presents an investigation into the synchronization of three identical oscillators, emphasizing the role of small impacts in pairwise oscillator interactions. We demonstrate that the system achieves synchronization with a constant phase difference between consecutive oscillators. The analysis reveals that these synchronized states form an attractor with a basin encompassing the closure of entire space of initial conditions. Our methodology involves the construction of a two-dimensional nonlinear discrete dynamical system, relevant to a variety of weakly coupled periodic oscillators under symmetric mutual impacts. The discussion extends to the examination of oscillation amplitude variations in the synchronized state, specifically within the framework of interacting Andronov pendulum clocks, providing detailed insights into amplitude changes in the locked state.

11 Iryna Sushko

11.1 Regular and chaotic dynamics of a 2d financial market model

Author: Iryna Sushko and Fabio Tramontana
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We consider a two-dimensional discontinuous map F defined by two linear functions, where one acts in the partition between two (parallel) discontinuity lines and the other one acts outside this partition. Map F described dynamics of a financial market model where three types of traders operate simultaneously: fundamentalists and chartists of two types, namely, trend followers and contrarians [1]. Our analysis shows that despite the linearity of the map components, its dynamics can be quite complex, with various, possibly coexisting attracting cycles and chaotic attractors. To understand how the overall bifurcation structure observed in the parameter space of the map is organized, we obtain analytically the boundaries of periodicity regions related to the simplest attracting period-n cycles, $n \geq 3$, with one point in the middle partition and $n-1$ points outside it. These boundaries can be related to border-collision bifurcations (when a point of the cycle collides with a discontinuity line) as well as to degenerate bifurcations (associated with eigenvalues on the unit circle). We show also some elements of period-adding and period-incrementing bifurcation structures (see [2] for details related to these structures in one-dimensional discontinuous piecewise increasing maps known as Lorenz maps) for which the cycles mentioned above are basic.

References

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12 Jochen Jungeilges

12.1 Stochastic sensitivity analysis of a 2D discontinuous financial market model with heterogeneous agents

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Based on [1] we specify a stochastic financial market model in which fundamentalists and two types of chartists, i.e. trend followers and contrarians, trade simultaneously. The asset price process evolves according to a two-dimensional discontinuous map defined by two linear functions. One of the functions operates in the partition between two (parallel) discontinuity lines, whereas the other acts outside this partition. Despite its linearity, the deterministic skeleton exhibits complex dynamics with potential coexistence of various attractors. We review existence and stability conditions for the actual - and the virtual – fixed point of the map as well as for the simplest cycles. We concentrate on a parametrization for which all attractors (cycles) coexist with the attracting actual fixed point.

Our analysis of the stochastic system focuses on the sensitivity of fixed points and cycles to additive noise. All results are obtained by combining the stochastic sensitivity function (SSF) approach, a mixture of analytical and numerical techniques, due to [2] with concepts and techniques employed by [1] in the study of the deterministic 2D discontinuous financial market model. In particular, we elaborate on conditions in terms of the behavioral model parameters under which noise induced transitions between selected attractors (fixed point, cycles) occur. From an economic point of view, we demonstrate that a fairly simple financial market model with heterogeneous agents is able to produce complicated boom-bust dynamics as well as intermittencies that are observed on real financial markets.

References

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13 Jorge Buescu

13.1 Iterative functional equations: from dynamics to fractal interpolation

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In this talk we shall survey our work in the area of iterative functional equations. These are closely related to dynamics, where in the autonomous case they function as topological conjugacies. We generalize to the non-autonomous case, studying the basic problems of existence, uniqueness and continuity of solutions in terms of compatibility conditions of the defining equations and constructing an explicit solution formula. We develop extensions of Barnsley fractal interpolation and provide to real-life problems in biodiversity.

14 Justyna Jarczyk

14.1 Complex-valued solutions of an iterative functional equation of finite order

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This is a report on the research made jointly with Witold Jarczyk.

Fix a positive integer $n \geq 2$ and a number $a \in (0, +\infty]$. Let $f_1, \ldots, f_n$ be continuous selfmappings of the interval $(0, a)$ summing up to the identity function:

$$\sum_{j=1}^{n} f_j(x) = x, \quad x \in (0, a).$$

Given an $(n-1)$-th root $\omega \in \mathbb{C}$ of unity and a complex number c, and defining $\psi_{\omega,c} : (0, a) \to \mathbb{C}$ by $\psi_{\omega,c}(x) = \omega \exp(cx)$, we see that

$$\begin{aligned} \psi_{\omega,c}(x) &= \omega \exp\left(c \sum_{j=1}^{n} f_j(x)\right) = \omega \prod_{j=1}^{n} \exp\left(c f_j(x)\right) \\ &= \frac{\omega}{\omega^n} \prod_{j=1}^{n} \psi_{\omega,c}\left(f_j(x)\right) = \prod_{j=1}^{n} \psi_{\omega,c}\left(f_j(x)\right) \end{aligned}$$

for all $x \in (0, a)$. Therefore $\psi_{\omega,c}$ satisfies the functional equation

$$\psi(x) = \prod_{j=1}^{n} \psi\left(f_j(x)\right).$$

During the talk I am going to prove that under some assumptions also the converse is true.

15 Lenka Rucká

15.1 Genericity of distributional chaos in non-autonomous systems

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The subject of this talk is the solution of two open problems concerning distributional chaos in non-autonomous discrete dynamical systems stated in [1] and [2]. In the first problem it is wondered if the limit function of convergent non-autonomous system with positive topological entropy is DC2. We show that the answer to this problem depends on the given metric and can be both, positive or negative. In the second open problem it is wondered if to be DC1 is a generic property of non-autonomous systems. The answer to this question is positive for convergent systems on the interval and negative for convergent systems on the Cantor set, regardless of what metric is used.

References

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16 Michal Málek

16.1 Sharkovsky's contribution to the understanding of omega limit sets

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Oleksandr Mykolayovych Sharkovsky made significant contributions to the theory of dynamical systems, particularly in the context of closed intervals. His most notable work is the Sharkovsky theorem, which deals with the ordering of periods that can simultaneously occur in one-dimensional dynamical systems. However, his contributions extend beyond this theorem to other areas of dynamical systems theory, including omega limit sets, combinatorial dynamics, atractors and many others.

We will focus on his contribution to the theory of omega limit sets for the case of dynamical system generated by a continuous maps on compact interval.

17 Paweł Pasteczka

17.1 Extension theorem for simultaneous q-difference equations and some its consequences

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Given a set $T \subset (0, +\infty)$, intervals $I \subset (0, +\infty)$ and $J \subset \mathbb{R}r$, as well as functions $g_t : I \times J \to J$ with t's running through the set

$$T^* := T \cup \left\{t^{-1} : t \in T\right\} \cup \{1\}$$

we study the simultaneous q-difference equations

$$\varphi(tx) = g_t\left(x, \varphi(x)\right), \qquad t \in T^*,$$

postulated for $x \in I \cap t^{-1}I$; here the unknown function φ is assumed to map I into J. We prove an Extension theorem stating that if φ is continuous [analytic] on a nontrivial subinterval of I, then φ is continuous [analytic] provided $g_t, t \in T^*$, are continuous [analytic]. The crucial assumption of the Extension theorem is formulated with the help of the so-called limit ratio R_T which is a uniquely determined number from $[1, +\infty]$, characterising some density property of the set T^*. As an application of the Extension theorem we find the form of all continuous on a subinterval of I solutions $\varphi : I \to \mathbb{R}$ of the simultaneous equations

$$\varphi(tx) = \varphi(x) + c(t)x^p, \qquad t \in T,$$

where $c : T \to \mathbb{R}$ is an arbitrary function, p is a given real number and $\sup I > R_T \inf I$.

These results were obtained jointly with Witold Jarczyk.

References

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18 Peter Raith

18.1 Inequalities for the topological entropy of topologically transitive piecewise monotonic maps

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Consider a piecewise monotonic map $T : [0,1] \to [0,1]$, this means there exists a finite partition $\mathcal{Z}$ of $[0,1]$ into pairwise disjoint open intervals with $\overline{\bigcup_{Z\in\mathcal{Z}} Z} = [0,1]$ such that $T\big|_Z$ is continuous and strictly monotonic (increasing or decreasing) for every $Z \in \mathcal{Z}$. Moreover, assume that for every $Z \in \mathcal{Z}$ the map $T\big|_Z$ is differentiable. Note that T need not be continuous at the endpoints of the elements of $\mathcal{Z}$.

Suppose that $\lambda > 1$ and that T is topologically transitive. If $\sup |T'| \leq \lambda$ and $\sup_I |T'| < \lambda$ for a nonempty open interval $I \subseteq [0,1]$ then $h_{\text{top}}(T) < \log \lambda$. Analogously, if $\inf |T'| \geq \lambda$ and $\inf_I |T'| > \lambda$ for a nonempty open interval $I \subseteq [0,1]$ then $h_{\text{top}}(T) > \log \lambda$. Examples are presented showing that these results are not true for continuous maps $T : [0,1] \to [0,1]$.

19 Roman Hric

19.1 Inheritance of minimality from continuous to discrete time systems

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Iteration theory is inherently based on using discrete time to iterate a process step by step. However, we often need to deal with phenomena that occur in continuous time. One of the most natural approaches to modelling a continuous-time system by a discrete-time system is to perform a time discretisation of the former to obtain the latter. The question remains, however, which properties of the original systems are preserved in this process.

One of the most fundamental properties of a dynamical system is topological minimality, the property that all the orbits of the system are dense in its state space. As we show in the joint work with Ľubomír Snoha [1], minimality is "usually" preserved in the process of time discretisation. I will present some results from the paper related to this problem.

References

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20 Saber Elaydi

20.1 Global Asymptotic Stability of Evolutionary Periodic Ricker Competition Models

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We investigate the global dynamics of discrete-time phenotypic evolutionary models, both autonomous and periodic. We developed the theory of mixed monotonemaps and applied it to show that the positive equilibrium of the autonomous evolutionary Ricker model of single and multi-species is globallyasymptotically stable. Then we extend this result to the corresponding evolutionary Ricker model with periodic parameters.

21 Tatyana Perevalova

21.1 Sensitivity analysis for attractors in a 3-person consumption network

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The dynamic model of interdependent consumer behavior [1] is generalized to the 2-commodities, n-person case, embedded in a stochastic environment. In particular, we focus on the dynamics of the n = 3 person case. While individuals' own consumption experience plays only a weak role in the formation of their preferences, the past consumption of (a subset of) peers in the network strongly affects the choice of preferences. In this case, a 3-dimensional, non-linear, stochastic system describes the consumption dynamics evolving in the space of budget shares. We demonstrate the coexistence of attractors, characterize their stochastic sensitivity following [2] and analyze noise induced transitions.

References

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22 Teresa Faria

22.1 Nonautonomous Nicholson equations with mixed monotone nonlinearities

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A general nonautonomous Nicholson equation with multiple pairs of delays in *mixed monotone* nonlinear terms is studied. Sufficient conditions for permanence are given, with explicit lower and upper uniform bounds for all positive solutions. Under an additional condition on the size of some of the delays, and by using an adequate difference equation of the form $x_{n+1} = h(x_n)$, we show that the solutions are globally attractive. In the case of a periodic equation, a criterion for the existence of a globally attractive positive periodic solution is provided. Some examples illustrate the results.

References

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23 Witold Jarczyk

23.1 Iterative roots of piecewise monotonic functions extended from the characteristic interval

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This is a report on the paper [Aequationes Math. 97 (2023), 1107-1128] written jointly with Veerapazham Murugan and Murugan Suresh Kumar from India, and Justyna Jarczyk.
We consider the problem of finding iterative roots of order less than the number of forts of continuous piecewise monotonic functions with nonmonotonicity height greater than 1. We present sufficient conditions to extend iterative roots of piecewise monotonic functions from the characteristic interval which determines the behaviour of the function under iteration.

24 Zdeněk Kočan

24.1 Solutions of a Cauchy-Gołąb-Schinzel type functional equation on a cone

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Let $\mathcal{C}$ is a convex cone in a real linear space and M be a nonempty set endowed with a binary operation $*$. We determine solutions of the functional equation

$$f(x+y) = f(x) * f(h(x)y) \quad \text{for} \quad x, y \in \mathcal{C} \tag{2}$$

in the class of pairs of functions (f, h), where $f\colon \mathcal{C} \to M$ and $h\colon \mathcal{C} \to [0, \infty)$. We extend the results in [1], where $\mathcal{C} = [0, \infty)$.

References

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25 Ziyad Al-Sharawi

25.1 An alternative to the Jury's stability algorithm

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Studying eigenvalues of matrices or local stability in dynamical systems involves examining the roots of polynomials. A polynomial is considered Schur stable if its roots cluster within the open unit disk. In this presentation, we introduce a novel method for analyzing Schur stability. Our process involves establishing a partial order in $\mathbb{R}^n$ and using the embedding technique to identify restrictions on the polynomial coefficients. The obtained restrictions ensure that the zeros of the polynomial are clustered within the open unit disk. This approach provides us with an alternative to the widely used Jury's stability algorithm. We provide several examples that illustrate the effectiveness of our technique.

References

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